Identification of Output-Error (OE) Models using Generalized Spectral Decomposition

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Problem statement and setting

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Identify the **dynamic (difference equation) model** from input data and output **measurements** using **generalized spectral decomposition** (GSD)

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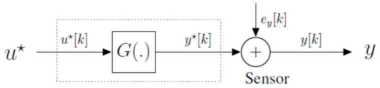

Figure 1: OE Model Setup

$$y^\star[k] + \sum_{i=1}^{n_a} a_i y^\star[k-i] = \sum_{j=D}^{n_b} b_j u^\star[k-j], \qquad \eta = \max(n_a, n_b)$$

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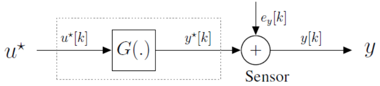

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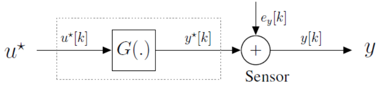

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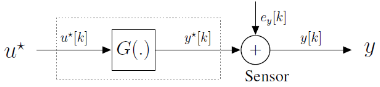

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Existing Methods (Ljung, 1998)

- **Prediction Error Minimization (PEM)** : minimize n-step ahead prediction error
- **Steiglitz-McBride (SM)** : minimizes the mean-square error between system and model outputs
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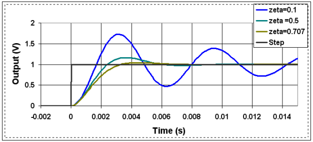
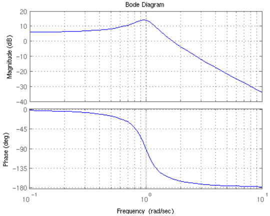
Shortcomings

All these approaches deliver unbiased and consistent parameter estimates but for right choice of model structure - delay, order

Model Structure Determination

- Pre-Estimation
  - User specified using domain / process knowledge
  - Nonparametric analysis : step response, frequency domain methods





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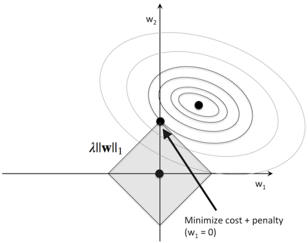

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  - User specified using domain / process knowledge
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- During Estimation
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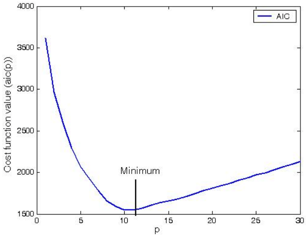
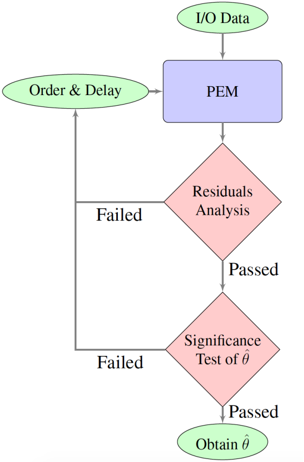
- Post Estimation
  - Residual Analysis
  - Information criteria such as AIC or BIC



Comparison of Methodology



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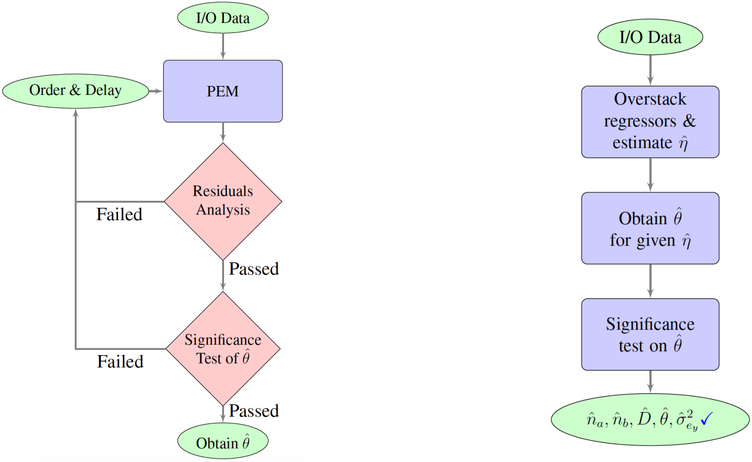

Figure 2: Comparison of Existing & Proposed Method

Dynamic Iterative PCA (Maurya et al., 2018)

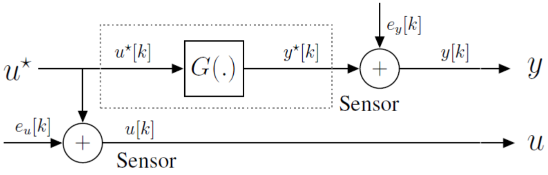

Figure 3: EIV Setup

DIPCA iteratively estimates

- noise covariance matrix

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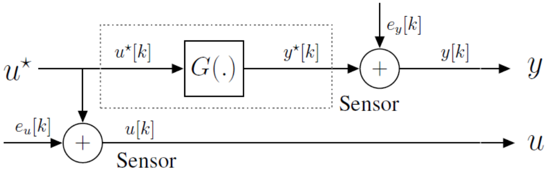

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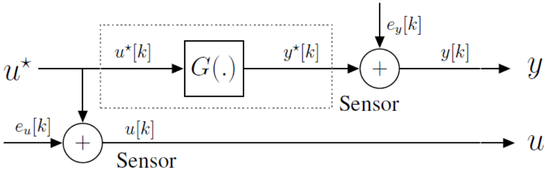

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- equation order, $\hat{\eta}$
- parameter vector, $\hat{\theta}$

PCA for identification: Quick review (Jollife, 2002)

Suppose $\mathbf{z}^\star[k] \in \mathbb{R}^{M\times 1}$ are **instantaneously** related through d **linear** constraints

$$\mathbf{A}\mathbf{z}^\star[k] = 0, \qquad \mathbf{A} \in \mathbb{R}^{d\times M}, \ \text{rank}(\mathbf{A}) = d \tag{1}$$

Then, given N noise-free observations of $\mathbf{z}^\star[k]$ in $\mathbf{Z}^\star = \{\mathbf{z}^\star[k]\}_{k=0}^{N-1} \in \mathbb{R}^{N\times M}$, the following results fall out from the SVD (PCA) of $\mathbf{Z}^\star$ under $d < M$

1. rank$(\mathbf{Z}^\star) = M - d$, i.e., the last d singular values, $\sigma_{M-d+1} = \cdots = \sigma_M = 0$.
2. The **right singular vectors** corresponding to the d zero singular values provide **a basis for the null space** of $\mathbf{Z}^\star$, i.e.,

$$\bar{\mathbf{A}} \triangleq \bar{\mathbf{V}} = \begin{bmatrix} \mathbf{v}_{M-d+1} & \mathbf{v}_{M-d+2} & \cdots & \mathbf{v}_M \end{bmatrix}^T = \mathbf{T}\mathbf{A}, \qquad \det(\mathbf{T}) \neq 0 \tag{2}$$

PCA & Variants

$$\mathbf{Z} = \mathbf{Z}^\star + \mathbf{E}, \qquad \mathbf{Z}^\star \mathbf{A}^T = \mathbf{0}$$

PCA produces **unbiased** estimates only in **homoskedastic** case, meaning

$$\boldsymbol{\Sigma}_\mathbf{e} = \sigma_e^2 \mathbf{I}$$

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Iterative PCA (Narasimhan and Shah, 2008)

- **Core idea** : Transform **heteroskedastic** problem to **homoskedastic** case by **scaling** with $\Sigma_\mathbf{e}^{-1/2}$, i.e., $\mathbf{z} \to \tilde{\mathbf{z}} \triangleq \Sigma_\mathbf{e}^{-1/2}\mathbf{z}$.

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- **Result** : Eigenvalues of the (sample) covariance matrices are **shifted by unity**!

$$\lambda(\mathbf{S}_{\tilde{\mathbf{z}}}) = \lambda(\mathbf{S}_{\mathbf{z}}) + 1, \qquad \mathbf{S}_{\tilde{\mathbf{z}}} = \frac{1}{N}\mathbf{Z}^{T}\mathbf{Z}$$

Dynamic PCA (Ku et al., 1995)

Idea: Apply *static* PCA to the matrix of lagged measurements $\mathbf{Z}$, i.e., treat the dynamic relation as a **static constraint on lagged variables**.

$$\mathbf{Z} = \begin{bmatrix} y[k] & \dots & y[k-L] & u[k] & \dots & u[k-L] \\ y[k+1] & \dots & y[k-L+1] & u[k+1] & \dots & u[k-L+1] \\ \vdots & \dots & \vdots & \vdots & \dots & \vdots \\ y[N-L] & \dots & y[N-L] & u[N-L] & \dots & u[N] \end{bmatrix}$$

Underlying relation: $\boxed{\mathbf{Z}^\star \mathbf{A}^T = 0}$

DPCA requires **exact stacking** i.e. $L = \eta \implies$ Order has to be known

Optimal only for homoskedastic cases i.e $\sigma^2_{e_u} = \sigma^2_{e_y}$

Dynamic Iterative PCA (Maurya et al., 2018)

Noise Covariance Matrix (Σ_e) is available

- Sufficiently **over-stack** the lagged input-output variables.
- **Scale** the data $\mathbf{z} \rightarrow \tilde{\mathbf{z}} \triangleq \Sigma_{\mathbf{e}}^{-1/2}\mathbf{z}$.
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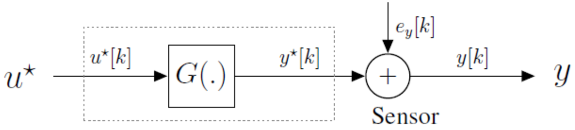
Noise Covariance Matrix (Σ_e) is NOT available

Estimate $\boldsymbol{\Sigma}_e$ by **maximizing the likelihood of residuals** from the assumed model.

Revisiting Classical Problem



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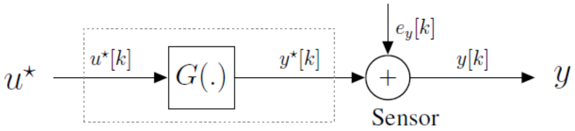

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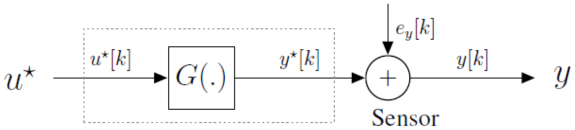

- **Scaling** ?
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- **Eigen-value shift** ?

For known order & noise variance

Proposition

DIPCA algorithm minimizes the **weighted TLS** cost function which could be derived from **generalized spectral decomposition**

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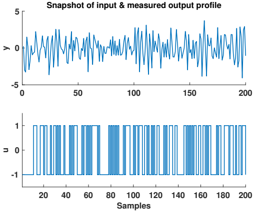
$$y[k] = y^\star[k] + e_y[k], \quad \text{var}(e_y) = 0.23 \quad \text{s.t SNR} = 10$$



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$$\begin{bmatrix} 2.63 & -0.6 & -1.23 & 1.2 \\ -0.60 & 2.63 & -0.6 & 0 \\ -1.23 & -0.6 & 2.63 & -0.02 \\ 1.2 & 0 & -0.02 & 1 \end{bmatrix} \mathbf{V} = \begin{bmatrix} 0.23 & 0 & 0 & 0 \\ 0 & 0.23 & 0 & 0 \\ 0 & 0 & 0.23 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{V\Lambda}$$

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Spectral Decomposition using QZ algorithm

$$\lambda_{min} = \mathbf{0.97} \qquad \hat{\theta} = \begin{bmatrix} 1 & \mathbf{0.4015} & \mathbf{0.5994} & \mathbf{-1.1907} \end{bmatrix}^T \quad \checkmark$$

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Eigenvalue shift theorem doesn't hold but zero eigenvalues of *noise-free* data maps to unity generalized eigenvalues.

Generalized Case

Order, delay and output noise variance are **unknown**

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where, $\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma_{e_y}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$

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$$\mathbf{S_{z^\star} v} = (\lambda - \sigma_{e_y}^2) \boldsymbol{\Sigma}' \mathbf{v}$$

1. $\boldsymbol{\Sigma}'$ is a **constant**
2. Instead of **unity**, look for **equal eigenvalues**

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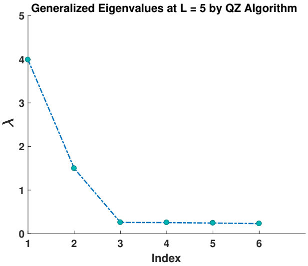
$$\mathbf{S_{z^\star} v} = (\lambda - \sigma_{e_y}^2)\boldsymbol{\Sigma}' \mathbf{v}$$



1. $\boldsymbol{\Sigma}'$ is a **constant**
2. Instead of **unity**, look for **equal eigenvalues**

Order, $\hat{\eta} = L - \hat{d} + 1 = 5 - 4 + 1 = 2$ ✓

Generalized Case

Order, delay and output noise variance are **unknown**

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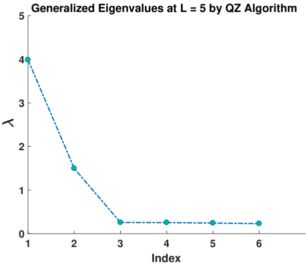

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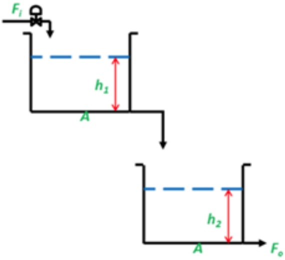
Model, $\hat{y}[k] + \underset{\pm(0.019)}{0.4005}\,\hat{y}[k-1] + \underset{\pm(0.018)}{0.6018}\,\hat{y}[k-2] = \underset{\pm(0.036)}{1.1976}\,u[k-1]$ ✓

Two non-interacting tank system



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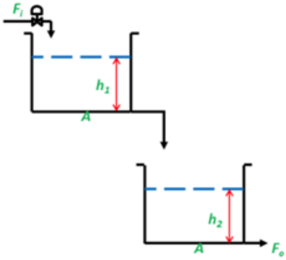

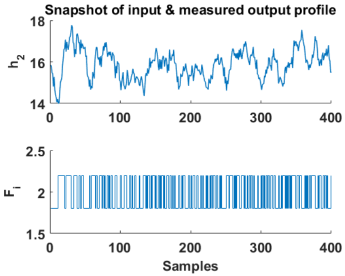

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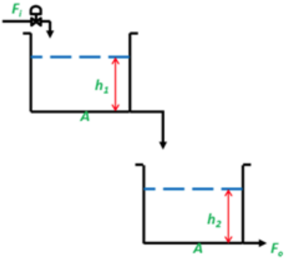

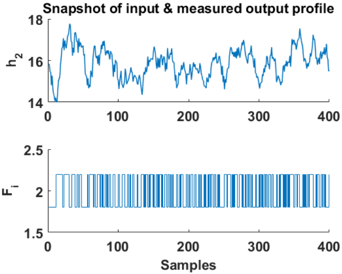

$$\frac{dh_1(t)}{dt} + \frac{Cv_1}{A_1}\sqrt{h_1(t)} = \frac{1}{A_1}F_i(t)$$
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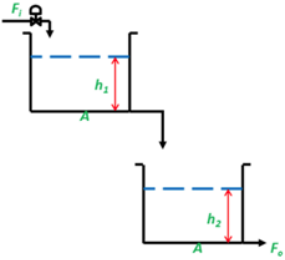

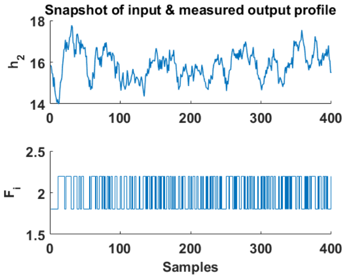

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Cv_1	Cv_2	A_1	A_2	h_{1ss}	h_{2ss}
1.8	0.5	2.4	1.2	1.23	16

Two non-interacting tank system

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Last 6 eigenvalues for L = 4

$$\Lambda = \begin{bmatrix} 2.03 & 0.0795 & 0.0457 & 0.044 & 0.0432 \end{bmatrix}$$

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$$G(s) = \frac{0.2813}{s^2 + 0.3896s + 0.0176}$$

Discretization under ZOH assumption

$$G(z^{-1}) = \frac{0.44z^{-1} + 0.33z^{-2}}{1 - 1.41z^{-1} + 0.46z^{-2}}$$

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- Generalized framework which could handle **both EIV and classical case** with different model structures.

Future Work

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  - Break up the given system into individual SISO systems using signal conditioning, i.e., work with *partial* covariance matrices.

Bibliography

- Jollife IT. Principal component analysis. 1986.
- Ljung, Lennart. "System identification." Signal analysis and prediction. Birkhäuser, Boston, MA, 1998. 163-173.
- Narasimhan, Shankar, and Sirish L. Shah. "Model identification and error covariance matrix estimation from noisy data using PCA." Control Engineering Practice 16.1 (2008): 146-155.
- Ku, Wenfu, Robert H. Storer, and Christos Georgakis. "Disturbance detection and isolation by dynamic principal component analysis." Chemometrics and intelligent laboratory systems 30.1 (1995): 179-196.
- Maurya, Deepak, Arun K. Tangirala, and Shankar Narasimhan. "Identification of Errors-in-Variables Models Using Dynamic Iterative Principal Component Analysis." Industrial & Engineering Chemistry Research 57.35 (2018): 11939-11954.

Acknowledgment

Various images in the PPT were borrowed from

- https://en.wikipedia.org/wiki/
- https://emojiterra.com/thinking-face/
- https://knowyourmeme.com/
- https://www.slideshare.net/

QZ Algorithm

The QZ algorithm is numerical method for solving generalized eigenvalue problem

$$\mathbf{Av} = \mathbf{\Lambda Bv} \tag{3}$$

without performing matrix inversion $\mathbf{B}$.

1. The idea is to transform (3) to
$$\mathbf{QAZy} = \mathbf{\Lambda QBZy}, \qquad \text{under} \quad \mathbf{v} = \mathbf{Zy} \tag{4}$$
where $\mathbf{Q}$ and $\mathbf{Z}$ are unitary matrices such that $\mathbf{QAZ}$ and $\mathbf{QBZ}$ are upper triangular
2. Eigenvalues can be computed from the diagonals of the triangular form. Eigenvectors can be computed by transforming back the eigenvectors of triangular problem with $\mathbf{Z}$