Hyperedge Prediction using Tensor Eigenvalue Decomposition

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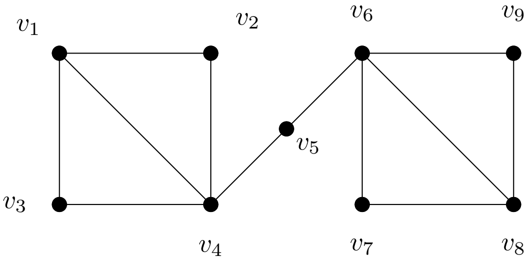
Indian Institute of Technology Madras, India

Hyperedge Prediction

Objective

Given a **k-uniform undirected hypergraph** $G = (V, E)$, predict the **new hyperedges** which are most likely to be formed.



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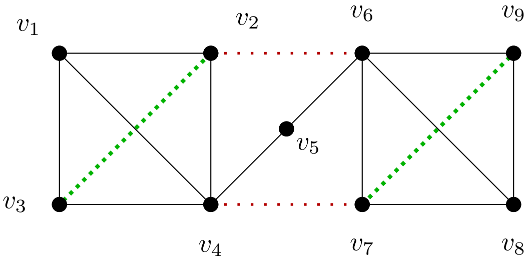

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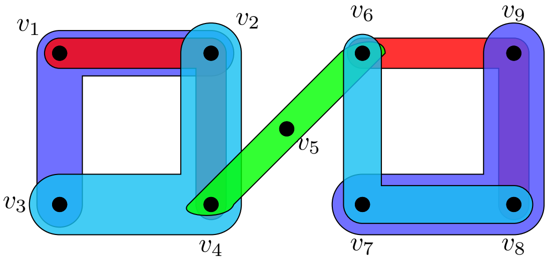
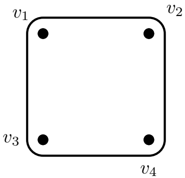


Existing Methods

Hypergraph Reduction

Reduce a given **hypergraph** H



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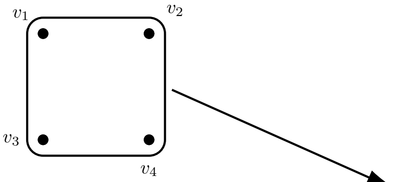
Hypergraph Reduction

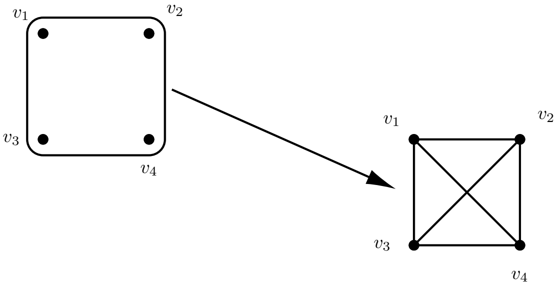
Reduce a given **hypergraph** H **to graph** G using **clique expansion**



Existing Methods

Hypergraph Reduction

Reduce a given **hypergraph** H **to graph** G using **clique expansion** and analyze the candidate hyperedges using **matrix representation**.



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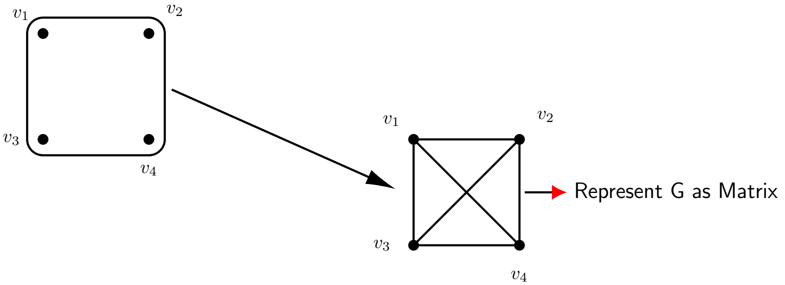

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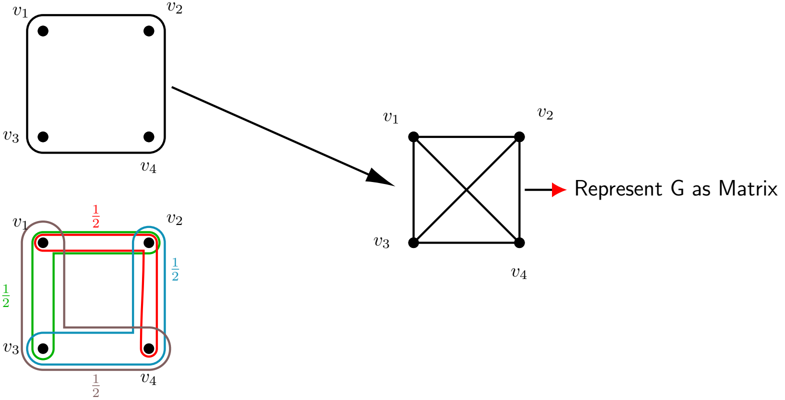

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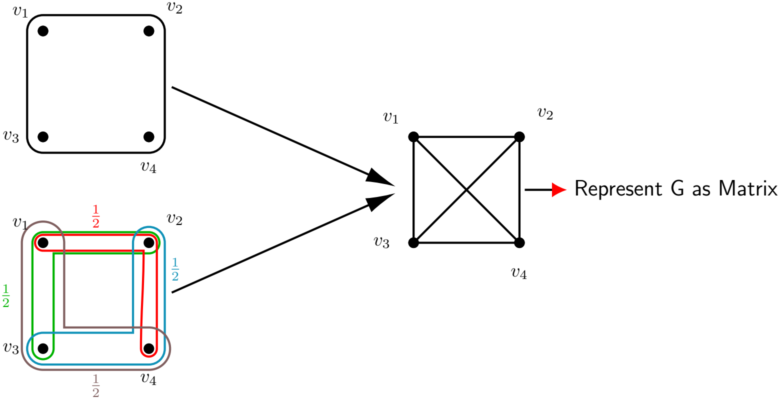
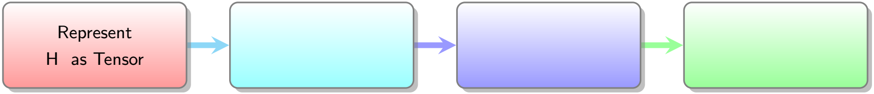


Proposed Algorithm

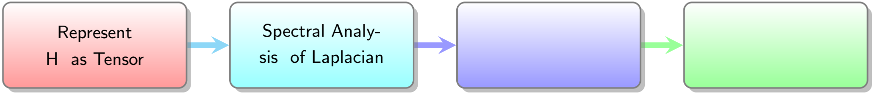


A natural representation of hypergraphs is a k-order n-dimensional tensor $\mathcal{A}$, which consists of n^k entries:

$$a_{i_1 i_2 \ldots i_k} = \begin{cases} w_{e_j} \frac{1}{(k-1)!} & \text{if } (i_1, i_2, \ldots, i_k) = \{e_j\} \quad e_j \in E \\ 0 & \text{otherwise} \end{cases}$$

It should be noted that $\mathcal{A}$ is a "*super-symmetric*" tensor.

Proposed Algorithm



The Laplacian tensor $\mathcal{L}$ is defined as:

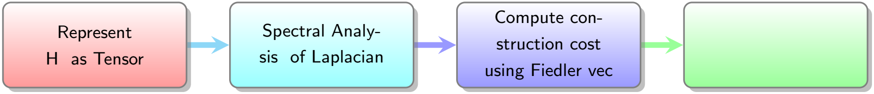
$$\mathcal{L} = \mathcal{D} - \mathcal{A}$$

Spectral decomposition using

$$\mathcal{L}\mathbf{x}^{k-1} = \lambda \mathbf{x}$$
$$\mathbf{x}^T\mathbf{x} = 1$$

where $(\lambda, \mathbf{x}) \in (\mathbb{R}, \mathbb{R}^n \backslash \{0\}^n)$ is called the Z-eigenpair.

Proposed Algorithm



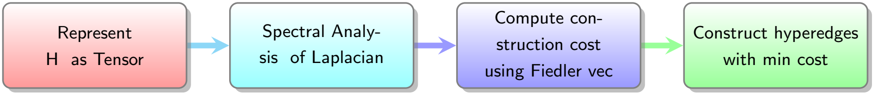
Construction cost for a new edge e_j:

$$l_{e_j}(\mathbf{x}) = w_{e_j}\left(\sum_{i_k \in e_j} x_{i_k}^k - k \prod_{i_k \in e_j} x_{i_k}\right)$$

$$= w_{e_j} k \left(\underset{i_j \in e_j}{\text{A.M}}\left(x_{i_k}^k\right) - \underset{i_j \in e_j}{\text{G.M}}\left(|x_{i_k}|^k\right)(-1)^{n_s}\right)$$

where $n_s = |\{i_j : x_{i_j} < 0\}|$, A.M and G.M denote arithmetic and geometric means, respectively.

Proposed Algorithm



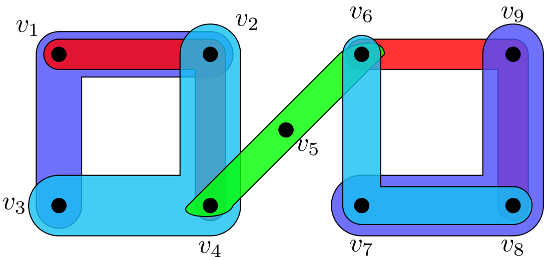
Construction cost for a new edge e_j:

$$
\begin{aligned}
l_{e_j}(\mathbf{x}) &= w_{e_j}\left(\sum_{i_k \in e_j} x_{i_k}^k - k \prod_{i_k \in e_j} x_{i_k}\right) \\
&= w_{e_j} k \left(\underset{i_j \in e_j}{\text{A.M}}\left(x_{i_k}^k\right) - \underset{i_j \in e_j}{\text{G.M}}\left(|x_{i_k}|^k\right)(-1)^{n_s}\right)
\end{aligned}
$$

where $n_s = |\{i_j : x_{i_j} < 0\}|$, A.M and G.M denote arithmetic and geometric means, respectively.

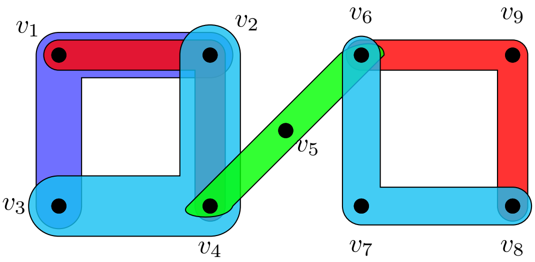
Example Hypergraph

Given the hypergraph H



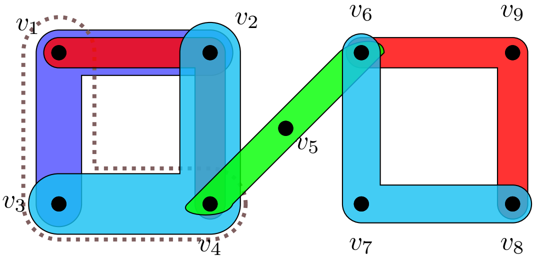
Example Hypergraph

Remove the hyperedge $\{7, 8, 9\}$ and predict new hyperedges



Example Hypergraph

First most likely hyperedge among 78 potential hyperedge: $\{1, 3, 4\}$



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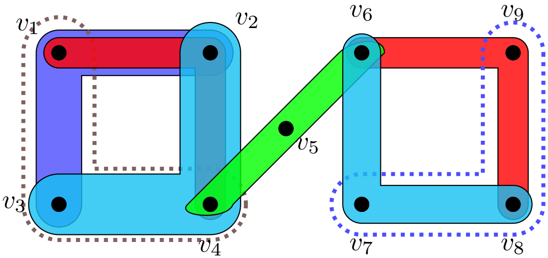

For More Information

Hyperedge Prediction using Tensor Eigenvalue Decomposition

Venue: Summit 9, ground floor of Egan Center (555 W 5th Ave)

Time: 11:30 AM to 12:00

Bibliography

- Qi L, Luo Z. Tensor analysis: spectral theory and special tensors. Siam; 2017 Apr 19.
- Banerjee A, Char A, Mondal B. Spectra of general hypergraphs. Linear Algebra and its Applications. 2017 Apr 1;518:14-30.